Double Your Kinetic Energy: Disappearance of the Classical Limit in Quantized Classical Energy Mechanics

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Abstract

Quantum concepts were first introduced by Max Planck and Albert Einstein in the early 1900's and efforts were soon underway to develop a description based on classical mechanics. Unable to successfully apply classical mechanics to quantized phenomena, quantum mechanics was developed instead. Certainty and definitiveness were no longer possible at the small length and energy scales, and only probabilities could be determined. Albert Einstein was profoundly troubled by this aspect of quantum mechanics and asserted there were hidden variables. The discovery of those variables (and constants) was recently announced. The new variables and constants have made possible the use classical mechanics to describe various aspects of light absorption and emission in a certain and deterministic manner. The classical limit has been pushed to the lowest possible energy levels for light, where it disappears altogether for those aspects. In addition, a well-grounded understanding of classical mechanics allows one to easily derive the new universal constants for light from first principles of position, time, and mass. The classical limit was in many respects an artifact of the hidden variables and constants. It remains to be seen if electron kinetics and the remaining transformations of light also yield themselves to the calculations of classical mechanics.

Introduction

Quantum concepts were first introduced by Max Planck and Albert Einstein in the early 1900's. Their hypothesis - that the energy of atom/electron oscillations and the light emitted and absorbed by atoms is quantized into small discrete amounts - was initially met with great skepticism. Subsequent experimentation eventually provided significant data that seemed to confirm their hypothesis. Efforts were soon underway in several countries to develop a description for these new concepts using classical mechanics. Planck's work had not provided a unit of energy for light, however, in the same way that Robert Millikan's work had provided the unit of charge for electricity. In Planck's quantum formula energy was variable, based on the product of an action constant and frequency. Although it seemed reasonable to expect that classical mechanics should be able to provide a correct description of physical processes at very small length and energy scales, work on the mechanical description for those processes was hampered by this dependence of the energy quanta on frequency, and the lack of an isolated unit quantity of energy.

After twenty years of failed attempts to explain quantized phenomena, two (2) new forms of mechanics were developed: Heisenberg's matrix mechanics and Schrödinger's wave mechanics. These new "quantum" mechanics could provide a mathematical framework for the low energy electron kinetics and light-related phenomena, however they were unable to provide the degree of certainty and definitiveness provided by classical mechanics in terms of position, time, and momentum, and instead were limited to finding probabilities. The efforts to apply classical mechanics to electron/light kinetics succeeded only at very high energy levels. This high energy level region where classical and quantum mechanics appeared to merge was deemed the "classical limit". Above the classical limit mechanics could be applied with reality and certainty, while below the limit uncertainty reigned.

Albert Einstein was profoundly troubled by this aspect of quantum mechanics, and was particularly vocal that quantum mechanics must therefore be an incomplete theory. He believed that the reason quantum mechanics could deal only in probabilities was because something was missing – "hidden variables". The discovery of Einstein's hidden variables (and constants) was announced recently at meetings of Materials Science and Technology 2009, SPIE Optics and Photonics, and the American Physical Society.[1-6] The new variables and constants have opened doors to more realistic interpretations of quantized phenomena, and have allowed development of new mechanistic models for previously unexplained physical phenomena related to the absorption, emission and transformation of light.[7-10]

This new work has revealed that it is now possible to use classical mechanics to describe various aspects of light absorption and emission, and that the "classical limit" in those regards was an artifact of the restricted degrees of freedom which had been imposed by the absence of the missing variables and constants. These limitations disappear in a classical framework with use of the new variables and constants. The classical limit has been pushed to the lowest possible energy levels for light, where it disappears altogether for those aspects. This extension of classical concepts down into the smallest energy realms of light has eliminated the need for a specialized mechanics divorced from the experience of every-day life. In fact, a well-grounded understanding of classical mechanics allows one to easily derive the new universal constants for light from first principles of position, time and mass.

For the ease of the gentle reader in quickly grasping the bold declarations just made, it will undoubtedly be helpful to first briefly review some of the new variables and constants, and secondly to explore some of the classical framework surrounding our concepts of energy. With this information well in hand, the reader will more easily follow the derivation of the new universal constants for light using simple classical mechanics, and gain an instant appreciation of the absence of any real "classical limit" between the quantum scale of small energies and the "classical" scale of large energies. Once in possession of this new understanding, the reader may begin enjoying the sense of greater certainty and determinacy, which is obtained from this more realistic and complete treatment of quantized classical energy mechanics.

The New Variables and Constants

Max Planck's (1858-1947) famous black-body radiation work, in which he derived the mathematical function describing changes in the distribution of light at various frequencies emitted by an object solely as a result of a change in its temperature, included some novel concepts and hypotheses.[11] One of those was the quantization of energy. Unfortunately, due to an inadvertence in his mathematics and his seeming desire to avoid the use of a time variable, Planck derived his famous constant *"h"* not as an isolated quantity of energy (mass x distance^2 divided by time^2), but as an *"action"* constant (i.e., energy x time, which is mass x distance^2 divided only by time).[1-4]

This inadvertence took place after a particularly embarrassing interchange Planck had with Ludwig Boltzmann (1844-1906) regarding Planck's consideration of time in his theoretical electromagnetic work. Planck thereafter adopted the methods of Wilhelm Wien (1864-1928) to convert the experimental black-body data, from time dependent energy measurements, to energy density measurements seemingly *"independent"* of time. In reality however, Wien's conversion procedure actually fixed the time variable at "one second", and then cancelled out the variable altogether. This made it *seem* as though the energy density measurements were independent of time. The energy measurements were still quite dependent on time, however, even though time did not appear as a separate variable in Planck's famous (and incomplete) quantum formula *"$E = hv$"*.

Upon restoring the hidden time variable, Planck's complete quantum formula is revealed:

$$\boldsymbol{E = \tilde{h}\, t_m v} \qquad [1]$$

where *"E"* is energy, *"$\tilde{h}$"* is Planck's energy constant, "t_m" is measurement time, and *"v"* is frequency. Of particular interest is Planck's energy constant, which is 6.626 X 10^{-34} Joules per oscillation of light: the small, fundamental quantity of *energy* hypothesized by Planck.[1] This energy quantum is constant and unchanging, and is independent of light's wavelength, frequency or "photon" energy.[*] This constancy of Planck's energy constant over a change in time or space means that the energy is conserved and the constant is thus a fundamental and universal constant. Just as electricity has a fundamental unit of charge, light has a fundamental unit of energy.

While we can now enjoy the many revelations provided by knowledge of this universal energy constant, early quantum pioneers were not so fortunate. One of the issues with which Louis De Broglie (1892-1987) struggled was the very lack of a fundamental energy quantum. In 1905, Albert Einstein (1879-1955) had proposed that, *"The mass of a body is a measure of its energy content; if the energy changes by L, the mass changes in the same sense by L / c^2 ... If a body emits the energy L in the form of radiation, its mass decreases by L / c^2."*[12] In his Ph.D. thesis, De Broglie adopted Einstein's suggestion that *"energy may be considered as being equivalent to mass, and all mass represents energy…we may regard material and energy as two terms for the same physical reality".*

Using Planck's incomplete quantum formula however, De Broglie was constrained to working with an energy variable – the product of action and frequency - rather than the fundamental energy quantum. Never-the-less, De Broglie gallantly tried to make sense of the paradox created by the fact that it was *"impossib[le] to consider an isolated quantity of energy"* and yet *"we have returned to statements on energy as fundamental and ceased to question why action plays a large role".* Without Planck's complete quantum formula, De Broglie was forced to conclude that it was necessary to always associate energy with frequency, rather than being able to consider an isolated quantity of energy, i.e., the fundamental unit of energy (6.626 X 10^{-34} J) carried by the elementary particle of light - a single oscillation of light.

Taking these limitations in stride, De Broglie proposed that since *"$E = hv_0$"* and *"$E = mc^2$"*, then, *"$hv_0 = mc^2$"*, and the variable rest mass of light is:

$$\boldsymbol{m_0 = h\, v_0 / c^2} \qquad [2]$$

De Broglie[13] declared this *"general formula ... may be applied to corpuscles of light on the assumption that here the rest mass m_0 is infinitely small….the upper limit of m_0…is approximately*

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[*] Photon energy is calculated using Planck's *incomplete* quantum formula, with its fixed time measurement of "one second". This calculation simply sums the energy of each individual oscillation into a total. The number of oscillations to be summed is determined by the frequency, *"v",* which indicates the number of sequential oscillations in "one second". The photon, rather than being a real physical particle, is an artifact of Planck's incomplete formula.

10^{-24} grams." This produced, of course, a veritable zoo of masses for light, given that De Broglie's mass was directly proportional to frequency, and the range of frequencies in the electromagnetic spectrum is infinite.

From our vantage point now, perched high atop the shoulders of De Broglie and Einstein, De Broglie's calculation for the rest mass of light has now been completed using Planck's complete quantum formula and the universal energy constant. One obtains as the rest mass for a single oscillation of light:

$$m_0 = 7.372 \text{ X } 10^{-51} \text{ kg/osc} \quad [3]$$

This figure is in close agreement and within the same order of magnitude as calculations by *Luo et al* of the upper limit of "photon" mass.† Light's oscillation rest mass is invariant and conserved over a change in frequency (time) or wavelength (space) and is thus another fundamental and universal constant.

De Broglie also used Planck's incomplete formula to derive the momentum for light:

$$\rho = m_0 c = h\upsilon / c = h / \lambda \quad [4]$$

finding that the momentum of light appeared to be directly proportional to its frequency, and thus inversely proportional to its wavelength "λ". Once again, De Broglie obtained another zoo - this time of momentum values, since the range of frequencies and wavelengths in the electromagnetic spectrum is infinite.

Taking full advantage of Planck's complete quantum formula and the new variables and constants, De Broglie's momentum of light has been definitively calculated:

$$\rho = m_0 c = \tilde{h}\, t_m \upsilon / c = 2.21 \text{ X } 10^{-42} \text{ kg m / sec per osc}^{\ddagger} \quad [5]$$

As with mass, the observant reader instantly recognizes that Planck's complete quantum formula provides a constant for the momentum of a single oscillation of light rather than a zoo of variables.

While the mass and momentum constants for light appear to be at odds with De Broglie's results, the reader should not despair. It will be recalled that masses and momenta are additive under the principles of classical mechanics. The masses and momenta of multiple oscillations are thus properly summed under a classical framework when multiple oscillations are measured in any particular time interval. When this is done for a "one second" time interval, which is the fixed value of the hidden time variable which De Broglie unknowingly used, the summed values yield the masses and momenta that De Broglie obtained. De Broglie was limited to using energy variables rather than the *"isolated quantity of energy"* over which he opined. His mass and momentum calculations were necessarily also variable as a result. The degrees of freedom which De Broglie sought are obtained with the energy constant, using classical mechanics at the lowest energy levels.

In summary then, Planck's quantum formula was missing the variable for measurement time "t_m", which obscured the nature of his constant as an *energy* constant, "$\tilde{h}$", and also obscured the true elementary particle of light, a single oscillation. After restoration of Planck's complete quantum formula, the energy constant was used to calculate both the constant rest mass and the momentum for the elementary particle of light. Mass, momentum and energy are all conserved and constant for light. What is *not* constant in regards to light is the *force* which it exerts upon absorption or emission from a body or material. This fact will be explained more fully below, when yet another of the

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† When *Luo et al's* upper limit of "photon" mass is converted to mass per oscillation, an upper limit of 4.32 X 10^{-51} kg/osc is obtained, well within the same order of magnitude as De Broglie's rest mass for a single light oscillation.

‡ Values of t_m = *1 sec/osc*, and υ = *1 osc/sec* can be used to obtain the momentum of a single oscillation.

hidden variables – the force of a single oscillation of light - is revealed. This explanation, however, requires a careful consideration of the classical concepts related to motion and energy, and so we shall turn to that topic next.

Classical Concepts Related to Energy and Kinetic Energy

The story of the energy of motion is one filled with controversy. Sir Isaac Newton (1643 – 1727) proposed in 1687 that the force carried by an accelerating body was equal to the product of its mass and acceleration. He distinguished this force of acceleration from the *effects* of an object in motion – the *vis viva* or "living force" of an object – which he asserted was the product of its mass and velocity. A few decades later Giovanni Poleni (1683–1761) of the University of Padua demonstrated that the "*Quantity of the Effect"* of the *vis viva* was proportional to the product of the mass and the velocity *squared,* putting him at odds with Newton's assertion. Leibnitz and Huygens soon followed in agreement with Poleni. This touched off a series of debates and disagreements as to whether the measurement of the effects produced by *vis viva* was properly represented as mass times velocity - or - as mass times the *square* of velocity.

According to Dutch physicist Willem Gravesande (1688 –1742), "[T]*he first Controversy about the Measure…[was] between Huygens and the Abbot Catalan…[and] Before the Controversy was ended, another arose between Leibnitz and the fame Abbot Catalan."*[14] Gravesande performed a series of meticulous experiments to test and measure the effects of *vis viva*, and weighed in firmly on the matter with Poleni, Huygens and Leibnitz. Gravesande declared that, *"If bodies by acting lose their whole Forces, the Effects follow the Ratio compounded of the Masses, and the <u>Squares</u> of the Velocities".*

Gravesande's work, was cited by the noted French Newtonian scholar, Emilie du Châtelet (1706 – 1749) in her 1740 work, "*Institutions Physiques".* Although she was an ardent disciple of Newton, she too agreed with Leibnitz based on Gravesande's firm experimental data, and asserted that *vis viva* was proportional to the velocity *squared*. Attacks on her work in the French Academy of Sciences soon followed, including those by her own research partner, Voltaire. Gravesande's painstaking experiments withstood all scrutiny, however, and eventually carried the day. Still devoted to Newton's work, du Châtelet spent her last years translating Newton's entire *Principia* into French, making the work easily accessible to other European scientists, such as the young Joseph Louis Lagrange.

Lagrange (1736 – 1813) developed an intense interest in mathematics in his late teens, and in his twenties began writing about mechanical issues using Newton's calculus. In his famous analysis of mechanics of 1788 (revised in 1811), Lagrange considered the *vis viva* of an undisturbed body moving at constant velocity, as opposed to a body subjected to an accelerating force.[15] He denoted the *vis viva* of such a body moving at constant velocity as "mv^2", based on *"the square of the distance that the body traverses during the instant dt"*.[§] For a many-bodied system, Lagrange wrote that the *"quantity* ($mv^2 + m'v'^2 + m''v''^2$ + *etc.)…expresses the <u>vis viva</u> of the entire system"*. Thus, according to Lagrange, the energy of a system at constant velocity "*results solely from the inertia forces of the bodies"*.

As for accelerations, Lagrange formulated the *"moment of force"* (i.e., energy) which caused a body to accelerate from a first velocity to a second velocity. The moment of force was a function

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[§] Some symbolic notation has been changed to make it consistent with modern usage. For example, the letter "*v*" was substituted for Lagrange's letter "u" to represent velocity.

not of inertia, but rather of the distance over which the force acted, i.e., *"Fδs"* (where *"F",* force is mass x acceleration, and *"s"* distance). Lagrange then considered the two quantities of effects – the *vis viva* of a body moving at constant velocity and the moment of force of an acceleration - in a dynamic equilibirum:*"The sum of these two quantities, when equated to zero, constitutes the general formula of dynamics…these forces equilibrate each other and the system is in equilibrium, but when the equilibrium does not hold, the bodies must necessarily move due to all or some of the forces which act on them."*

Lagrange's formulation of dynamic equilibrium in this simple and concise manner was premised on an assumption not well described in modern texts. Indeed, it appears to have been all but forgotten and is found only in a detailed reading of Lagrange's two (2) volume tome. For the ease of conveying the brilliant simplicity of his mechanical concepts, Lagrange assumed that the unit time for velocity, and the time interval in which an acceleration took place, were identical: "*assuming that for each accelerating force …the basic unit of time…[is] the same time if it moved uniformly"*. In other words, Lagrange assumed that the acceleration did not occur in a time interval which was small compared to the velocity unit time, meaning that the acceleration (or deceleration) could not be *"instantaneous"*. Lagrange assumed that an acceleration always occurred in a time period sufficiently long as to be equal to the "one second" unit time for velocity.

Thus Lagrange effectively fixed the acceleration time variable at "one second". This created an interesting boundary condition which limits the rote application of Lagrange's mechanical equations, to instantaneous quantized phemonena. If an acceleration (or deceleration) takes place in a time interval much shorter than "one second", this fact must be accounted for and the fixed time variable must be modified accordingly under the framework laid out by Lagrange.

Forty years later, Gaspard-Gustave de Coriolis (1792–1843) relied heavily on Lagrange's work in his popular engineering textbook on machines with levers, screws and tackle blocks, "*Calculation of the Effects of Machines"*.[16] Coriolis adopted Lagrange's elegant description of forces and equilibrium, stating, *"forces capable of producing equilibrium are forces that always cancel each other out"*. Likewise, Coriolis also assumed that accelerating forces act over a fixed time period of "one second" and explicitly excluded instantaneous effects, *"since consideration of these instantaneous forces is not necessary…[and] we will not make use of them."* Thus, instantaneous phenomena, such as the absorption or emission of a single oscillation of light, were outside the boundary conditions in both Coriolis' and Lagrange's works.

Coriolis also emphasized a particular aspect of Lagrange's dynamic equilibrium, for the convenience of engineers working with 18th century machines. "[T]*he mass times <u>one-half</u> the square of the speed…will introduce more simplicity into the statements of principles…since the factor '½(v^2/g)' is nothing more than the height from which a heavy body, in a vacuum, must fall so that it may acquire the speed 'v'…[T]he expression [½mv^2]…need be considered only as an abbreviated way to designate the product of the weight times the height due to the speed."*[**] The energy quantity "½mv^2" was thus used by Coriolis as a convenience in engineering applications for machine effects resulting from the constant force of gravity. Coriolis gave his abbreviated "weight times height of the fall" a new name, *kinetic energy*. He was very aware of the fact that his shorthand formula for kinetic energy did *not* apply to objects moving at constant velocity, stating that when *"the speeds have become the same…[the kinetic energy] becom[es] zero"*.

Coriolis' simplified descriptions of mechanics for engineers working in industry were immensely popular, and had a lasting effect on modern science and technology. Due to the

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[**] The constant *"g"* is gravitational acceleration. The object was assumed to be dropped from a position of rest with an initial velocity of zero.

widespread use of Coriolis' book however, Lagrange's "mv^2", for an object moving at constant velocity, faded into the dim recesses of mathematical esoterica. Soon scientists and engineers alike were of the conviction that the kinetic energy of an object moving at constant velocity was "½ mv^2", and the fixed "one second" time interval for acceleration was all but forgotten as well.

One gains a good sense for the evolution of these forgotten details in the mechanical physics textbook written by James Clerk Maxwell (1831-1879).[††] Maxwell first provided a clear and concise derivation for kinetic energy based on averaging the two different velocities which exist before and after a constant accelerating force:

> "The original momentum is *MV*, and the final momentum is *MV'*, so that the increase of momentum is *M(V'-V)*, and this, by the second law of motion, is equal to *FT*, the *impulse* of the force *F* acting for the time *T*. Hence
>
> $$FT = M(V' - V) \qquad \text{.........(1).}$$
>
> Since the velocity increases uniformly with the time [when the force is constant], the mean velocity is the arithmetical mean of the original and final velocities, or $\frac{1}{2}(V' + V)$.
>
> We can also determine the mean velocity by dividing the space *S* by the time *T*, during which it is described. Hence
>
> $$S / T = \tfrac{1}{2}(V' + V) \qquad \text{.........(2).}$$
>
> Multiplying the corresponding members of equations (1) and (2) each by each we obtain
>
> $$FS = \tfrac{1}{2}MV'^2 - \tfrac{1}{2}MV^2 \qquad \text{.........(3).}$$
>
> Here *FS* is the work done by the force *F* acting on the body while it moves through the space *S* in the direction of the force, and this is equal to the excess of $\frac{1}{2}MV'^2$ above $\frac{1}{2}MV^2$. If we call $\frac{1}{2}MV^2$, or half the product of the mass into the square of the velocity, the *kinetic energy* of the body at first, then $\frac{1}{2}MV'^2$ will be the kinetic energy after the action of the force *F* through the space *S*." (Emphases and brackets original)

Maxwell then summarized with a statement indicative of the forgotten details:

> "The *kinetic* energy of a body is the energy it has in virtue of being in *motion…*" (Emphases original)

Within a century Lagrange's energy of "mv^2" was deliberately excluded from the literature,[‡‡] and it was commonly being taught that the kinetic energy of a body moving at constant velocity is equal to "$\frac{1}{2}mv^2$" (although technically it has *no* kinetic energy at all). Texts now teach that a body moving at constant velocity has kinetic energy equal to the work it would potentially perform, were it to collide with another body. The body moving at constant velocity would have to come to rest in a constant deceleration in a time interval of at least "one second" for it to have kinetic energy of "$\frac{1}{2}mv^2$", however. If it does not come to rest and merely slows, the kinetic energy is "$\frac{1}{2}m(v' - v)^2$".

Coriolis' definition of kinetic energy was premised on the assumption that there are *no instantaneous* changes of velocity, because he fixed acceleration time equal to velocity unit time. The phenomena associated with the emission and absorption of light are *instantaneous* when compared to the time scales considered by Lagrange and Coriolis. Thus, the teachings of classical mechanics laid out by both Lagrange and Coriolis require an understanding of the principles they

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[††] *"Matter and Motion"*

[‡‡] See the book review in *Nature*, Vol 74, (No. 1072), 1890, p. 98, complaining that, *"the word force vive for mv^2 is still allowed to appear in these pages, in spite of all the recent efforts of Thomason and Tait, Maxwell and recent writers to banish it into oblivion."*

described, and an appreciation of the boundaries posed to a rote application of their equations to instantaneous events such as absorption and emission of light.

Albert Einstein's (1879-1955) theory of special relativity remedied the mechanical treatment of light in some regards, although for completely different reasons. (Einstein, like De Broglie, had no *"isolated quantity of energy",* so he had to redefine the meaning of simultaneous events in time.) None-the-less, his relativistic equation for the conservation of energy and mass used "mass times the square of velocity", i.e., *"mc^2"* for light rather than one-half the value:[17]

> "Classical mechanics required to be modified before it could come into line with the demands of the special theory of relativity. For the main part, however, this modification affects only the laws for rapid motions, in which the velocities of matter are not very small as compared with the velocity of light. We have experience of such rapid motions only in the case of electrons and ions; for other motions the variations from the laws of classical mechanics are too small to make themselves evident in practice. … the kinetic energy of a material point of mass *m* is no longer given by the well-known expression
>
> $$m \frac{v^2}{2}$$
>
> but by the expression
>
> $$\frac{mc^2}{\sqrt{1 - (v^2 / c^2)}}\text{"}$$

which reduces to Einstein's famous equation, *"$E = mc^2$"*, similar to Lagrange's *"mv^2"*.

Derivations and Calculations

§ 1. Derivation of Force Exerted By Light

Force is the product of mass and the change in velocity "*during the instant dt"* when the velocity changes:

$$F = m\, v / dt \qquad [6]$$

Typically, the unit time for velocity and acceleration are identical, however they need not be, and in the case of light, generally are not due to the instantaneous nature of light emission and absorption. The "*instant dt"* for a light wave traveling in a vacuum is the time that elapses between crests, i.e., between the leading edge and then the trailing edge passing a measuring point, known as the time period, *"τ"* for light. Time period for light is the inverse of its frequency, *"υ"*, ("$\upsilon = 1/\tau$").

When a single oscillation of light encounters a massive body and is absorbed, rather than being reflected by the body, it adds its mass to the mass of the body. In other words, the mass of the light oscillation comes to rest in the body and its forward translational movement halts. The time period over which this instantaneous change takes place is the time period *"τ"* of the oscillation. Hence it can be seen that the force of a single oscillation of light is:

$$F_0 = m_0\, c / \tau \qquad [7]$$

Because of the reciprocal relationship between time period and frequency, this can also be written:

$$F_0 = m_0\, c\, \upsilon \qquad [8]$$

§ 2. Light's Oscillation Force is Variable and Dependent on Time Period (and Frequency)
Consider three different exemplary time periods $\tau_{1\text{-}3}$, such that:

$$\tau_1 >> \tau_2 >> \tau_3 \qquad [9]$$

$\tau_1 = 1.03 \text{ X } 10^{-3}$ sec/osc	>>	$\tau_2 = 5.00 \text{ X } 10^{-10}$ sec/osc	>>	$\tau_3 = 3.33 \text{ X } 10^{-16}$ sec/osc (UV)
(Radio frequency, 1 X 10^3 osc/sec)		*(MW frequency, 2 X 10^9 osc/sec)*		*(UV frequency, 3 X 10^{15} osc/sec)*

The force which can be exerted by a single oscillation associated with each of the above three time periods is determined according to Eq. 7.:

$$F_1 = 2.21 \text{ X } 10^{-39} \text{ N} \quad << \quad F_2 = 4.42 \text{ X } 10^{-33} \text{ N} \quad << \quad F_3 = 6.63. \text{ X } 10^{-27} \text{ N}$$

The force exerted by a single oscillation of light varies with time period and frequency. The shorter the time period over which the energy quantum/rest mass of a single oscillation of light is absorbed, the greater the force that light oscillation will exert on the absorbing mass. Higher frequency oscillations will exert more force, oscillation per oscillation, than lower frequency oscillations.

§ 3. Derivation of Energy Absorbed or Emitted as Light
Energy is the product of force and the distance "s" over which the force is exerted. The force of an oscillation of light is distributed along its wavelength. The distance "s" over which the force is exerted is thus the wavelength "λ" of light, and the energy of a single oscillation of light is:

$$E_0 = F_0 \; s = m_0 \, c \, \lambda \, / \, \tau \qquad [10]$$

Using frequency in place of time period this becomes:

$$E_0 = F_0 \; s = m_0 \, c \, \lambda \, \upsilon \qquad [11]$$

The quotient, *"λ/τ"* (or the product, "*λ υ"*) is equal to the ratio expressed in the constant speed of light, *"c"*. Although the time periods and wavelengths for individual oscillations are infinitely variable, their *ratios* are constant and identical. Hence, no matter what the wavelength and time period (or frequency) associated with any oscillation of light, the ratio of its wavelength to time period is constant. As a result, the product of an oscillation's force and distance *"λ",* includes the constant and invariant space-time coupling constant, the speed of light*, "c".*

The relationship for light's constant oscillation energy of Eq. 11., above, reduces to Planck's energy constant, *"$\tilde{h}$" :*

$$E_0 = m_0 \, c^2 = \tilde{h} \qquad [12]$$

Discussion

The initial discovery of Planck's complete quantum formula revealed the fundamental energy constant, *"$\tilde{h}$",* and the measurement time variable. Subsequent completion of De Broglie's calculations provided fundamental constants for mass and momentum. Recent work has now demonstrated that the energy constant for light can be derived using classical mechanics.

The application of classical mechanics requires the ability to determine with certainty the position, time, and mass of a body. Historically, this was a rather simple matter when dealing with large objects such as cannon balls and pendulums. As the application of classical mechanics moved to smaller and smaller scales, however, the ability to determine position and time with certainty pushed the limits of experimental technology. Consider the theoretical work of Maxwell on electromagnetic radiation, which had to wait several decades for experimental confirmation by Hertz. Hertz not only confirmed Maxwell's theoretical waves, but also measured the wavelength, time period, and phase of the electromagnetic oscillations he had experimentally generated, and in the process determined the position and time of the light oscillations with great certainty.

A decade later, Einstein formulated his principle of mass and energy equivalence for quantized light. Experimental advances occurring many years later have since proven that Einstein's "modification of classical mechanics" is firmly grounded in reality, and the bombardment of massive bodies with protons, X-rays and γ-rays bears testament to the equivalence and conservation of mass and energy. Before experimental techniques had even caught up yet, however, De Broglie applied Einstein's hypothetical rest mass for light in his formulations on the momentum of light. It was then another decade before the momentum of light was confirmed experimentally.

Using the position, time, and mass of light described in a classical framework by these earlier scientists, it is now possible to describe and calculate the force exerted by light when it is absorbed by a body with mass. This accomplishment requires an understanding of the assumptions that went into many of the classical mechanics equations. In particular, detailed reading of both Lagrange's and Coriolis' mechanics reveals that ***a second time variable was fixed at a value of "one second".*** In the case of Planck's measurement time variable, the fixed and hidden value appears to have been inadvertent, while in the case of Lagrange and Coriolis, the fixed acceleration time interval was deliberate and accompanied by the appropriate caveats as to the boundary conditions for such a fixed time variable. Pursuant to the selection of the proper *instantaneous* acceleration time interval for absorption of an oscillation of light, the formula for the force of light has been derived. This light force formula indicates that the force exerted by light is variable, depending on light's time period, and reveals yet another of Einstein's hidden variables – the force variable for light.

Experimental confirmation that light does indeed exert an actual accelerating *force* on matter, as opposed to simply possessing momentum, was recently demonstrated. In 2008, Weilong *et al* demonstrated that the force of light leaving the end of a silica filament causes the filament to recoil, just as the force of a bullet accelerating and leaving the barrel of a gun causes the gun to recoil.[18] As of this writing in July 2010, Liu *et al* have just demonstrated that the force produced by light on the arms of a nanomotor, produces enough torque to cause the rotation of a microdisc 4,000 times larger in volume than the motor.[19] The rotation occurs maximally at resonant frequencies where the greatest absorption of light's mass and applied force takes place. As Millikan observed, *"absorption [of electromagnetic energy] is due to resonance (and we know of no other way in which to conceive it…)".* [20] At resonant frequencies, where maximal accelerating light forces were exerted, the velocity of the rotating disc was the greatest.

With knowledge of the position, time and force variables for light, the constant energy of a single oscillation of light may be derived under a classical mechanics framework from first principles. Energy is calculated as the product of force applied over a distance. Consistent with the carefully described caveats of Lagrange and Coriolis, the distance over which the force of light is exerted is the wavelength of the oscillation. (Note that this distance of light's force exertion should not be confused with *travel* distance of the light waves: a 1 GHz oscillation traveling across the laboratory will exert the same amount of force, as a 1 GHz oscillation traveling across the galaxy. Proper use of action constructs such as the Hamiltonian must necessarily take this fact into consideration.) The fundamental constancy of light's conserved oscillation energy, over a change in time or space, is thus easily derived using classical mechanics.

The quantized classical energy mechanics described above do not admit of any limit on the treatment of absorption and emission of light, nor is any limit apparent in the use of the smallest known quantized energy in Planck's energy constant, *"$\tilde{h}$"*. Although only quantum mechanics were previously thought to account for certain quantized phenomena, such as absorption and emission of light, these low energy phenomena – indeed, these ***sub-photonic*** phenomena - can now be accounted for with classical mechanics. The classical limit does not appear to exist for these phenomena.

When Niels Bohr wrestled with these same issues a century ago, determined to find a classical description for Planck's varying energy, he was unaware that Planck had inadvertently fixed the hidden time variable in his quantum relationship to a period of "one second". It also appears that he was unaware of a ***second*** critical time variable which was ***also fixed*** at a value of "one second" - the variable acceleration time interval. In the visible and ultraviolet light regions in which Bohr was trying to classically model the electronic spectrum of the hydrogen atom, the acceleration time interval was much less than a trillionth of a second. Yet Bohr was in the frustrating position of having to work with formulae in which two different time variables had been fixed at an entire second, a value fifteen orders of magnitude larger than the oscillation time periods under study, and which yielded energy values a thousand trillion times larger than the fundamental energy quantum.

It is small wonder then, that after working on the problem for several years, Bohr resigned himself to the fact that he could not successfully apply classical mechanics at low energies, and that they could only be used for large energies. The large energies where classical mechanics appeared to work, were equivalent to the total energy of the trillions of oscillations associated with the fixed "one second" time intervals of the hidden time variables. Bohr's hapless conclusion about large energies induced him to propose the existence and limitations of a "classical limit".[21] In reality, however, Bohr's classical limit was nothing more than an artifact of the missing constant for the "isolated quantity of energy" and of the fixed and hidden time variables. Next steps in this new work will be to see if electron kinetics yield themselves to classical mechanics, just as light absorption and emission phenomena have.

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